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RESEARCH ARTICLE

Financial Stability and Financial Distortions: A Saddle Path Equilibrium between the Crowding Out Effect and the Financial Accelerator

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Abstract

Various methods of modeling based on stress testing and macrofinancial modeling have been adopted in the literature on financial stability assessment and all display merits as well as drawbacks. The literature on the subject matter of financial stability assessment is still in its infancy and requires novelties with respect to methodologies adopted in the assessment process. The eviction effect or the crowding out effect and the financial accelerator are two issues that are compromisingly involving for financial stability, economic policy making and the nexus between the macroeconomic imperfections and financial imbalances. As far as interactions between key determinants of financial stability are concerned, there is a longstanding dissensus on the impact of the crowding out effect on financial stability as it is an issue intertwined compromising public policy and financial matters whereby the crowding out effect is found to be acting like an actor partially offsetting and slowing down the negative effects of the financial accelerator on financial stability in what we call a saddle path unstable equilibrium following the tendency of the trend of the effect of the financial accelerator on financial stability temporarily then abruptly turning the tendency of the effect of the financial accelerator on financial stability sequentially to mitigate instability partially. This raises concerns about a novel method to assess financial stability and the likely merits of modeling financial stability taking into consideration these two items of commensurate relevance and importance to highlight two main economic effects namely the double dividend theory in presence of distortions and the close relationship of economic and financial matters in face of the hindrance of providing a clear insight into interactions and cross effects among determinants of financial stability. The main scope of going deeper into the intertwined relationship between the eviction effect and the financial accelerator is to better fathom financial stability while at the same time waiving the drawbacks of conventional methods of financial stability assessment that always impair features of commensurate relevance to herald specific concern about microfinancial considerations or macrofinancial considerations and in some instances prioritizing rare innovations or contrariwise neglecting them roughly. The proposed saddle path equilibrium modeling shows clearly the cross effects and interactions of relevant features composing financial stability assessment while at the same time including key microeconomic features relevant to systemic risk in a comprehensive way.

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The approach highlights indeed the role played by equity and credit in affecting financial stability. It provides impetus on the effect of government bonds and securities between the credit market and the stock market sectors that makes improvement in stock market sector trading of government securities or attract ability to banking placement or incur draining loanable funds through increase in stock market profitability and deteriorates credit market financial stability by the demand pull effect through the effect of liquidity risk on credit risk in the face of limited tolerance to overall risk incurred by banks and the search for yield motivation of banks. The effect on financial stability through the Stock market sector or what we call the saddle path unstable equilibrium between crowding out effect and the financial accelerator constitutes the cornerstone of our research and will be thoroughly analyzed and scrutinized throughout the mathematical proofs provided in the research referring to the polynomial distributions of risk, the gaussian pivot for multivariate discrete functions, the saddle path unstable equilibrium and the reference to sensitivity analyses and constitutes the novelty provided by this research in face of the drawbacks of available measurement methods displayed in the literature of relevance for the subject matter.

Key Words: *Crowding out effect; Financial accelerator; Double dividend theory; Financial stability; Saddle path; Equity; Borrowing; Polynomial distribution of risk*

1. Introduction

Financial stability has been defined in the literature with consenting arguments. Dissensus arises only for the issue of measurement or in other words the methodological approach to adopt to better grasp the key features of prominent relevance of its fathoming and their interactions that are intrinsic to their cross relations.

K Bollen, et al. [1] defined financial stability as: "It is a condition where the financial system is capable of performing well all of its normal tasks and where it is expected to do so for the foreseeable future. From this viewpoint financial system stability requires the principal components of the system including financial institutions markets and infrastructures to be jointly capable of absorbing adverse disturbances. It also requires that the financial system is facilitating a smooth and efficient allocation of financial resources from savers to investors, that financial risk is being assessed and priced reasonably accurately and that risks are being efficiently managed". According to them: "Financial stability has also an important forward-looking dimension: inefficiencies in the reallocation of capital or material shortcomings in the pricing of risk can by laying the foundations of future vulnerabilities compromise future financial system stability and therefore economic stability".

Although the issue of financial stability measurement is relatively new there is a vast amount of literature dealing with the issue. The available literature adopts various methodologies that constitute the building blocks of the paradigmatic purview of the sources of instability and tries to depict them in order to provide the adequate modeling of this issue.

As far as our methodological approach is concerned, we sketch to the mainstream definition of financial stability to show that in a new Keynesian framework it is of paramount importance to take into considerations sources of imperfections and disruptions or vulnerabilities and their interactions in order to measure financial stability. Public Finance and equity of corporations can play comparable roles with respect to banking credit to the private sector that constitutes a key source of financing fueling private investment and therefore crowding out effect and items such as agency costs and information asymmetries

might evict private investment compromisingly. The idea of the financial accelerator is that small shocks to the macroeconomy amplify through agency costs and information asymmetries to result in big consequences at the financial level. Small shocks affect directly net worth of corporations and thereby the arbitrage between internal and external borrowing and lead to a shock of more commensurate depth in the credit market and eventually following the first one in the equity market. A financial accelerator is defined by Federal Reserve Board Chair Bernanke, et al. [2] as a means by which developments in financial markets amplify the effects of changes in the economy. In a new classical framework, the effect of the crowding out effect is clearly of prominent importance and is through its reference to the credit sector obviously linked to the accelerator effect somehow.

We propose to analyses the dynamics of the relationship between the crowding out effect or the eviction effect and the leverage effect or the financial accelerator and will show that it is subject to unstable equilibriums of the saddle path type and that these two key macrofinancial phenomena are constituting key features helping to provide a novel measurement of financial stability following a modeling based on advanced mathematical tools like polynomial dispersion of risk and saddle path unstable equilibrium using Gaussian pivot to highlight minimax regions of stability or instability solving thereby the issue of jump effect that most models fails to model.

We will prove that these jump effects are indeed intrinsically depictable from the very non linearity of relationships among the constituting building blocks measuring financial stability. The computational tool adopted is using sensitivity analyses where the elasticity is assumed to grasp at best the salient impact effect measuring the phenomena of eviction and financial acceleration.

We will then go more in depth into the analysis to suggest some recommendations in the face of the clear longstanding ineffectiveness of conventional methods of measurement of financial stability and show that the novel method can provide impetus and an additional tool of commensurate relevance for policy makers in the face of the hindrance of referring to other methods of measurement to motivate policy making.

Referring to the new Keynesian framework, The financial accelerator in itself is an obstruction to financial stability when financial stability is defined as the ability of the financial system and the real economy to efficiently allot resources assess and manage financial risks and eliminate relative price movements of real or financial assets that will affect monetary stability and generate financial imbalances that will result in shocks that inhibit self-correcting mechanisms preventing adverse events from disrupting the real economy or the financial system. Indeed, the phenomenon of financial accelerator creates a shock that inhibits self-correcting mechanisms between the real economy and the financial system.

As a matter of fact, it acts like a transmission mechanism that transmits net wealth shocks from the real sector to the financial sector through the banking sector and the agency cots due to the tradeoff between internal and external financing therefore abasing the self-correction of the economic system as the real economy shocks propagate to the financial sector prices volatility and fluctuations in the stock market of assets share prices.

Whereas, the crowding out effect although a friction in itself when coupled with the disruption of the financial accelerator can partially offset the detrimental effect of the financial accelerator on financial stability depending on the phase of the unstable equilibrium among them as will be shown and discussed more in depth later in the theoretical discussion.

Hence the objective of the paper is to show that in the new Keynesian framework gathering features salient to financial stability measurement that are distortions or imperfections can generate a partially offsetting effect of their negative impacts although the mechanism involves a complicated mathematical unstable equilibrium that helps fathom the shape of financial stability which is the saddle path type of equilibriums with reference to polynomial distribution of risk and sensitivity analyses.

2. Literature Review and Methods

There is a consensus in the referred to literature that the most practical challenge in implementing a framework for financial stability assessment revolves around the disciplining of the process of assessment or in other words grasping the key method of assessment.

According to K Bollen, et al. [1], "The main hindrance to financial stability assessment should be to determine whether the financial system is in a zone or corridor of financial stability or outside a zone or corridor of financial stability. Within the third category the financial system could be further judged to be in a position which self-corrective processes and mechanisms are assessed as being likely to the system back towards the corridor of stability or alternatively need prompt remedial and even emergency measures to reverse the instability". In the context of the IMF and FSAPs like recalled by those same authors, current practices are far from achieving the measurement of possibly nonlinear interaction between the financial and non-financial sectors of the economy. But, the implementation of macroeconomic stress-testing frameworks has undoubtedly advanced the development of internally consistent frameworks for assessing the resilience of financial systems to adverse disturbances.

According to Hoggarth, et al. [3], "Stress testing and macro econometric testing should be combined but display the limitation of potential second round effects that can herald and these models pay insufficient attention to macro-financial interaction". The novelty of our approach is that it uses the minimax optimization tool of a saddle path type of unstable equilibrium using Gaussian pivot in a saddle path type of analysis to find a way out of the insufficiencies of conventional modeling to grasp macro-financial interactions with reference to key features of systemic risk moments that show relevance of polynomial distribution further motivating saddle path equilibrium type of interlinkages among key components of financial stability assessment.

And in fact, the adopted methodology finds a saddle path type of equilibrium between the eviction effect and the financial accelerator effect that eventually helps plot the zone and corridor and outside zone and corridor of unstable equilibrium and with reference to the sensitivity's computations can provide an insightful tool of computation of these phenomena namely the crowding out effect and the financial accelerator. Although financial stability is mainly pertaining to monetary policy and is a central banking objective, it is considered to be indirectly affected by fiscal policy through the indirect effects of the crowding out effect on financial imbalances. In a New Keynesian framework, financial stability depends on the effects of fiscal and monetary policy on output, consumption, investment and inflation. Indeed, financial stability is closely related to prudential surveillance essentially tributary on the effects of credit risk and liquidity risk on exposure to systemic risk. The financial accelerator is mainly tributary on demand amplification effects rather than supply amplification effects. This means that the New Keynesian paradigm proponing demand amplification effects boosts investment demand while the supply shock is dampened.

In other words, for loan to deposit ratio, for instance, the demand shock boosts credit demand while the dampened supply shock dampenes saving which puts a pressure on liquidity risk thereby compromising financial stability. A New Keynesian definition of the financial accelerator in relation to its business cycle effects states that: "Conditions in financial markets and the economy may reinforce each other resulting in a feedback loop that produces a boom or bust despite the changes themselves being relatively small when examined individually. The magnification of the result is the financial accelerator".

A financial accelerator often comes out of the credit market and eventually works through to impact the economy as a whole. Financial accelerators can initiate and amplify both positive and negative shocks on a macroeconomic scale. The financial accelerator model was proposed to help explain why relatively small changes to monetary policy or credit conditions could trigger large shocks through an economy. For example, why does a relatively small change in the prime rate cause companies and consumers to slash spending even though it is a small incremental cost?

The financial accelerator theory proposes that, at the peaks of business cycles, the majority of businesses and consumers have overextended themselves to varying degrees. This means that they have taken on cheap debt to finance improvements or expansion to their businesses and lifestyles. This also means that they are extra sensitive to any changes in the credit environment, more so than they would be at other points in the business cycle. When the expansion portion of the business cycle comes to an end, this same overextended majority gets pinched by a poorer economy and tightening credit.

The idea of credit conditions influencing the economy is not a new one, but the Bernanke, Gertler, and Gilchrist model provided a better tool for guiding policy to take credit market impacts into effect. Even then, the financial accelerator model received very little attention until 2008, when Bernanke was at the helm of the Federal Reserve during a financial crisis that turned into the Great Recession. The financial accelerator model received a lot of attention as it provided a context for explaining the actions that the Fed was taking to minimize feedback loops or shorten their run time. This is one of the reasons why so many of the bailout measures, as they became known, were focused on stabilizing the credit markets directly through the banks. In the financial accelerator model, slowed credit causes a flight to quality. This means that weaker firms and consumers are abandoned and credit is offered only to stronger firms.

However, as more of these firms struggle with less consumer-driven buying, they also fall out of favor. This loop continues until much of the credit is squeezed out of the economy, resulting in a lot of economic pain. Bernanke used his knowledge of financial accelerators to try and limit the pain and shorten the amount of time that the U.S. economy suffered from tight credit conditions. The relevance of considering a saddle path type of equilibrium to qualify the relationship between the crowding out effect and the financial accelerator stems from the fact that their effects on each other are reverted which presumably ought to result in an unstable equilibrium. They are besides closely related because of the obvious diversity between public projects qualifications that are void of agency costs and information asymmetries and private projects management that contrariwise is subjected to excess information asymmetries and agency costs provided an arbitrage has been conducted about equity and external borrowing. In as much, crowding out effect affects considerably the arbitrage between external borrowing and equity financing for corporations and stands thereby at the forefront of the issue of the financial accelerator.

Their cross relation hence is key to better understanding how supply side economics and economic performance stimulation functions in reality. The crowding-out effect accentuates agency costs and worsens thereby financial acceleration. Information asymmetries have the opposite effect in the presence of crowding-out. Indeed, the more the supply of credit to the private sector is restricted, the more the quest for profitability forces the banking system to tolerate more asymmetries of information, because the cost of project feasibility studies becomes excessive. So, when asymmetries of information increase, the cost of arrangement decreases proportionately, and credit to the private sector becomes more accessible. Thus, in the quest for profitability by bank managers in the presence of information asymmetries, the effect of the detrimental crowding-out effect on the private sector is attenuated. In the case of public spending, information asymmetries are almost non-existent, since the incentives for free riding and adverse selection are insignificant for reasons of self-financing at the level of ministries, i.e., the financial commissions of ministries play the role of suppliers and applicants of funds. So, they manage to avoid adverse selection. But they don't need to free ride, since there are calls for tender for every acquisition opportunity.

According to macroeconomic theory in Keynesian economics growth can be stimulated through government spending meaning that increases in government spending deny feasibility of laisser faire as a strategic economic policy. This is mainly due to market imperfections. Therefore, Keynesian economics stipulate that as information asymmetries and capital imperfections increase meaning that the financial accelerator increases the crowding out effect decreases as government deficits help stimulate economic growth through endogenous investment.

According to New classical economics instead there is no room for increases in market imperfections and asymmetric information's so definitely the financial accelerator is small. Besides laisser faire is key so government spending should not increase and crowding out is big. The hypothesis of a strong correlation with inverted effects between financial accelerator and crowding out fits at best New Keynesian theory where market imperfections prevail along efficiency of government intervention through government deficit economic stimulation.

2.1. The financial accelerator

The concept of financial accelerator derives from informational asymmetries in the credit markets which drive a wedge between the costs of external and internal finance. Information asymmetries are defined by Mishkin [4] as being the free riding, the adverse selection and the moral hazard. These affect simultaneously the credit scoring and the risk of non-performing loans and reveal therefore costly in terms of banking profitability in particular and financial stability in general. The higher cost of external finance for corporations reflects the agency costs of lending under asymmetric information.

In such a situation, internally generated funds enable firms to reduce their demand for costly external finance. Hence a fall in a borrower's net worth or cash flow raises the demand for external finance and subsequently reduces investment. Given that negative shocks affect cash flows individually the effect of an initial shock to the economy is amplified. Small shocks might therefore create large cycles hence the term financial accelerator. A contractionary monetary policy shock now reduces investment spending through the traditional cost of capital effect and given that changes in interest rates affect corporate cash flows lowers a firm's cash flow and its ability to borrow. Consequently, monetary policy impulses are reinforced by cash flow effects.

In the relevant literature the Modigliani Miller theorem [5] states that: « financial structure is irrelevant to corporate investment decisions ». More recent research however has questioned the assumption of perfect substitutability of external and internal funds by pointing to the existence of capital market imperfections. The most common argument posits that asymmetric information and problems of contract enforcement led to the emergence of agency costs thereby driving a wedge between the cost of external and internal finance. As outside investors require a premium for unobservable uncontactable risks, external finance becomes more costly than internal finance. The financial accelerator measure is at best graspable by a sensitivity computation that better expresses the impact of real economic shocks on financial sector excess volatility and imbalances.

In other works, it is a sensitivity or impact analysis of the repercussions on the Stock market sector of an initial shock in the real economy.

The financial accelerator= elasticity of the principal component of the Stock market sector/ GDP growth= e(PCSI/GDP growth).

If the corporations are indifferent between internal and external financing then the financial accelerator is equal to one if not it exceeds or is below one as corporations have preference to equity financing or borrowing from the credit sector.

2.2. The crowding out effect

The study by Blanchard, et al. [6] for the postwar US economy reaches the conclusion that a shock in government spending causes output and private consumption to increase while private investment is crowded out.

Marattin, et al. [7] apply a panel vector autoregression approach to EU countries and find evidence of positive effects of increases in government spending on both consumption and investment.

The crowding out effect occurs when government spending evicts private investment. Because of substitutability between government borrowing and lending to the private sector government borrowing restriction affordability of the private sector to the credit marketing. As liquidity risk increases, the banking system with a certain tolerance to total risk decreases credit risk tolerable and thus decreases credit production to finance private investment. The situation is exacerbated by agency costs of the banking system that through the financial accelerator make internal financing more likely and inelastic in substitutability to external financing in the prevalence of both eviction and financial accelerator. Hence more stress is imposed on internal financing that makes stakeholders more imposing in terms of dividends and the burden of the cost of capital increases therefore considerably without affordability of arbitrage and bargaining power of corporate management in terms of internal and external financing. This definitely implies a negative balance sheet effect that results in decreasing private investment. When crowding out effect increases, agency costs for public entrepreneurs decrease due to the oligopolistic structure of the market of state entrepreneurs. Profitability from public borrowing decreases. Through the search for yield the credit market tries to compensate from the private sector. Agency costs for private sector decrease

- NPLs increase.
- Profitability shrinks.
- Borrowers' credit worthiness shrinks.

The increase in crowding out further exacerbates the financial accelerator in that the initial shock to corporations is more amplified by the acceleration to a financial negative repercussion or imbalance. The effect of crowding out on the financial accelerator is positive. Entrepreneurs with worst criteria of selection for government projects usually refer to banks in order to finance private projects.

While government projects require mostly heavily immobilized entrepreneurs that refer to banks to finance turnover and variable costs which do not imply a heavy credit allocation, lightly immobilized entrepreneurs that have a barrier to entry to government projects have a tendency to finance acquisitions of costly immobilization through banking resources. So, the consequences of non-performing loans for entrepreneurs in government projects are less involving for banks than credits to finance immobilization. Selection of government entrepreneurs restructures the destination of credit production to becomes more biased towards financing heavy infrastructure and machines for private entrepreneurs and turnover and flexible costs for government entrepreneurs. So as government expenditure increases So does the credit risk for private entrepreneurs. Meanwhile besides this aspect liquidity risk increases everything equal.

So definitely increases in government spending crowd out private investment through the increase of credit market imperfections, agency costs and information asymmetries towards the expectations of the risk entailed by the financing of purchasing of heavy industrial and technical machines as long as the heterogeneity of destination of banking credit is biased towards incertitude with increases in government spending. The incertitude does not essentially come from liquidity risk but from the restructuration of the quality of entrepreneurs because mainly those owning their own immobilizations are more likely to work for government projects than private investors financing their costly machines acquisitions through bank financing.

Therefore, the crowding out effect contributes in increasing the financial accelerator. The other way round shows that increases in agency costs and information asymmetries or the financial accelerator dumpers the crowding out effect. Now if there are more agency costs and information asymmetries arbitrage for internal financing increases. The private corporations find their way out of increases in external financing while corporations working in government projects are drained off of internal sources of financing. This is due to the fact that stakeholders are striving for high-risk high return activities contrariwise to banking governance.

Therefore, as financial accelerator increases government spending have the tendency to recruit less local private investors because they are less equipped with heavy immobilization but turns to foreign entrepreneurs and compromises exchange risk. This is likely to involve an ulterior burden of exchange rate risk and acts like a break for government spending.

Therefore, increases in the financial accelerator slow down government spending and by a consequence ease credit conditions for private investors thus decreases the crowding out effect. The wedge between internal and external financing widens, what makes risk premiums less affordable to the mainstream private investor that turns to internal financing through responding to stakeholder's expectations that are not likely to finance low risk low return projects like government projects but have a preference for high-risk high return projects like IT research and development microchips etc. In definitive if the financial accelerator increases government spending decreases.

But if government spending and by a way of consequence crowding out increases the financial accelerator increases. The crowding out measure is better grasped through the expression of the sensitivity of private investment to government spending which is e(PI/GS) expressing the impact of government spending increases on private investment or the eviction effect of private investment by government spending.

2.3. Financial stability the eviction effect and the financial accelerator: The intuition behind the main linkages

The financial accelerator which elasticity of the principal component of the Stock market sector/ GDP growth or in other words e(PCSI/GDP growth).and the crowding out measure which is better grasped through the expression of the sensitivity of private investment to government spending or e(PI/GS) expressing the impact of government spending increases on private investment or the eviction effect of private investment by government spending are fundamental to the methodological approach of this research with reference to the economic analysis.

According to this research qualitative matters should be grasped through impact effect that are accurately and implicitly expressed by sensitivities. But financial stability highlights an issue of commensurate importance which is the relevance of stability in financial matters that suggests a clear reference to the pertaining volatility control.

Volatility matters are closely tied in mathematics to standard deviation and variance. Henceforth, the purview of this research posits that financial stability is the variance of the addition of the crowding out effect and the financial accelerator that are basically two main distortions drawing the wedge between the real economy and the financial sector. This research disentangles itself from the mainstream literature that highlights the prominent role played by monetary policy matters and prudential surveillance matters whose main objectives is financial instability mitigation and attempts to pave the way for a theoretical consideration of the effect in a new Keynesian framework where imperfections and distortions are key of the role played by fiscal matters when closely bound to the transmission of macroeconomic shocks to the financial sector which is assessed by the financial accelerator in affecting the stability of the financial sector when stability is assessed through the mathematical variance. Henceforth, the basic expression for financial stability should be an inference on the variance of issues affecting commensurately the financial volatility and that are linked to investment the government spending or the role of the government with respect to fiscal policy in parallel to the transmission effect of macroeconomic shocks to the financial sector that are imbedded in the expression of the financial accelerator.

The investment item is considered to be key for financial stability items as it lies behind the instability generated by the credit sector as well as the excess volatility generated by the stock market sector and hence lies behind most of financial instability taking into consideration that the credit sector and the stock market sector are the two most prominent financial sectors. To provide a simple but robust expression the research provides the assertion that the financial stability should take the form of the variance of the cumulation of the distortions generated by the eviction effect and the financial accelerator and attempts to show that they generate an unstable equilibrium whereby the eviction effect plays a partial offsetting role following the double dividend theory to the prevailing distortion introduced by the financial accelerator. The logic of the double dividend theory is heralded by the reasoning that the second distortion introduced distorts in itself the distortion introduced by the first item so that that the initial negative effect is partially attenuated like for the environmental tax case further discussed in

the reference provided. Even though the conventional literature directly binds the financial stability issue to monetary policy and interest rate rules this research adopts the reasoning that new Keynesians that base their paradigm on imperfect competition convey a prominent role to supply side economics in contrast with the Keynesian economic paradigm that is founded on demand side economics and hence the investment aggregate plays a prominent role and should be key in determining a phenomenon like financial stability or financial instability to be more accurate.

In this respect monetary policy primary role highlighted in the relevant literature is relevant as long as it affects the credit sector whose primary target revolves around stimulating private investment and providing leakage to public investment. Hence in the analysis of financial stability the priority of threat to stability or instability should make investment play the most prominent role disrespectfully of the role played by monetary policy in affecting the credit sector behavior or the macroeconomy in general. But in the specific case where shocks are transmitted from the real sector to the financial sector a primary role bridging, he gaps between the eviction of private investment by government spending translating the role of the state and the transmission of shocks from the real sector to the financial sector translated by the financial accelerator have been considered to be a good fit of the key sources of instability to the financial sector.

In this respect, this new Keynesian framework as applied in this article has literally neglected the monetary policy item disrespectfully of its relevance in the conventional literature and the prominent role it plays in affecting output, consumption and credit allocation arbitrage as well as banking intermediation costs which are usually referred to in articles drawing a wedge between financial stability and monetary policy in the new Keynesian framework.

2.4. Effects of taxation on the relationship between the crowding out effect and the financial accelerator

First, as taxation increases, management of corporations flee from equity for which capital is taxable to borrowing which is not. The financial accelerator increases. As taxation increases government spending crowds out less private investment everything else equal which makes crowding out less in that government borrows less from the banking sector letting more liquidity for the banking sector and alleviating the supply push (cost push) that increases the borrowing rates. Therefore, with lower borrowing rates the financial accelerator slows down as corporations rely less on equity with cheaper external funding. The financial accelerator increases.

Therefore, as taxation increases the leverage effect increases more proportionately. By a way of consequence, the increase in taxation exerts a negative effect on the relationship between the crowding out effect and the financial acceleration in that it increases the negative correlation.

2.5. The new Keynesian framework, distrosions and the double dividend theory

2.5.1. New Keynesian paradigm and its relevance for the subject matter

The most adequate framework of the equilibrium between crowding out effect and leverage effect to be adopted is obviously the New Keynesian paradigm. As a matter of fact, the market structure of the credit market which is key to the relationship in unstable equilibrium is

monopolistic competition which added to supply side economics aspect of neoclassical economics makes the paradigm new Keynesian. This indeed reveals of prominent relevance for conducting a straightforward analysis of how do the financial accelerator effect interacts with the crowding out effect under the circumstance of borrowing rates being established through market power involved by monopolistic competition especially with key information asymmetries and agency costs imbedded in the notion of the acceleration.

The credit sector plays a prominent role in the new Keynesian paradigm and is key in affecting income and welfare and transmitting monetary policy effectiveness. Basically, the imperfect competition purview involved by the new Keynesian paradigm invalid for the credit sector which is usually either an oligopoly or monopolistic competition hence affecting the agency cost tremendously and the risk premium which in turn affect the arbitrage of corporations between internal and external financing and either accentuate or diminish the financial accelerator effect thereby affecting the transmission of real economic shocks to financial shocks and ending up by affecting financial stability.

The issue of monetary policy effectiveness for the new Keynesian purview is involved by the fact that financial stability is an objective of monetary policy which is claimed to be effective in the short run for staggered prices until prices adjust in the case of rational expectations in the long run and monetary policy becomes less effective meaning that the concern of mitigating financial instability becomes less permissible theoretically from the paradigmatic considerations without further scrutinizing the issue of the relationship between the financial accelerator the crowding out effect and financial stability.

2.5.2. Monetary policy and the nexus between crowding out effect and the financial accelerator

The Interaction between monetary policy and financial stability affects the equilibrium between crowding out effect and the financial accelerator in many respects. It is claimed to affect negatively financial stability. By affecting the money market rate, it can slow down the crowding out effect through accommodation and at the same time slowdown the substitution of external borrowing by raising equity and hence slowdown the financial accelerator provided information asymmetries remain the same which is not the case as whenever the money market rate slows down bankers will shift to risky entrepreneurs and hence information asymmetries will increase. Therefore, the financial accelerator decreases at a lesser extent than the crowding out effect. When money market rate increases the crowding out effect increases but at the same time the financial accelerator increases proportionately. This leads to the same saddle path and the same negative effect of the financial accelerator on financial stability Now the crowding out effect is claimed to dumpen or attenuate the negative effect of the financial accelerator on financial stability.

It follows that accommodation policy will result in the acceleration of the negative effect of the financial accelerator on financial stability, whereas the tightening policy will not affect the unstable equilibrium. There would be no stagnation unstable equilibrium between the crowding out effect and the financial accelerator but a gradual decline of the Saddle path equilibrium meaning that as the financial accelerator increases the crowding out effect at the beginning of the Saddle path increases and the converse effect of the crowding out effect slowed down by the financial accelerator would be more accentuated and the new equilibrium of the crowding out effect would be at a lower level. It would be like a downward sloping cascade or scale.

Therefore, the effect of monetary policy on the saddle path equilibrium between crowding out effect and the financial accelerator is nonlinear. Indeed, monetary policy deteriorates increasingly financial stability at accommodation policy and does not affect financial stability at tightening policy. In the interaction between financial stability and monetary policy the credit transmission mechanism acts as follows.

As credit supply increases there is an income effect whereby increase in credit supply increases responds to public sector needs for financing thereby affecting to a lesser effect eviction and here as financial stability enhancement attenuates crowding out effect as credit supply increases internal funding for corporations is attenuated in proportion and hence the super multiplier is attenuated. We are in a situation where both eviction and the super multiplier are attenuated. The substitution effect acts as catalyzer of eviction whereby the additional amount of credit dwells to the private sector due to search for yield.

The requirements of financing for the public sector acts as a demand pull for sources of financing intended to finance private sector credit the crowding out effect increases but as long as credit to the private sector increases the effect on financial accelerator is an attenuation. Thus, in this case the increase in the crowding out effect is accompanied with a decrease in the financial accelerator. In definitive the income effect of the credit transmission mechanism flattens the Saddle path shape of the unstable equilibrium between crowding out and the financial accelerator and makes it reach à turn point at a briefer timespan.

The rationale lying behind the positive effect of crowding out on financial stability although it is claimed to have a negative effect as it is a distorsion introduced by government intervention through budget deficits credit sector financing in the presence of financial acceleration which is a financial distorsion claimed to exert a negative impact on financial stability emanates from what we call the reverse effect of prevalence of several distorsions. The cumulative effect of prevalence of distorsions can reverse the negative effect of the existence of one distorsion. Several examples from the literature corroborate this finding among which the double dividends theory and the New Keynesian theory.

According to the double dividend's theory, the introduction of a green tax creates distorsions but can improve efficiency and reduce the negative effect of capital taxes and labor taxes. As stated in Goulder [8]: "the double dividend theory in its weak form claims that tax revenues from green taxes can be used to reduce other distortionary taxes like taxes on labor or capital and hence yield a second dividend". The idea of an excess benefit originates with Tullock (1967) and was put forward by several papers Nichols (1984) Terkla [9] and Lee and Misiolek [10].

In this regard we are concerned with the principal that the introduction of an additional distortion can reduce the negative effect of an outstanding distortion by partially offsetting its negative effect and not concerned with the green tax per se. Also, according to the New Keynesian theory, the prevalence of imperfect competition and sticky prices can leave room for beneficial intervention of the government that would not have been permissible had the expectations been rational and prices adjusting instantaneously without imperfect competition in Markets.

As stated in Laurence M Bal and David Romer [11] even if prices are perfectly flexible imperfect competition can affect the influence of fiscal policy in terms of the multiplier.

Recent work on New Keynesian models with convenience yields on other assets in particular Hagedorn [12] studying a model with uninsurable income risk and a risk-free rate some consumers are on their intertemporal Euler equation so that their marginal rate of substitution is not equated to their interest rate. Michaellat, et al. [13] assume that wealth is a separate argument in utility in addition to consumption.

The new Keynesian models taking the credit sector into account focus on asset pricing with investors who face liquidity or collateral constraints, dating back to Lucas [14] Kiyitaki and Moore, et al. [15] and Geanakoplos, et al. [16]. In this setting, agency costs are of prominent importance and fuel the financial accelerator we are covering in this article. This corroborates the idea that basically the implication of effective fiscal policy and information asymmetries or agency costs are related through a relationship we are looking to prove dependent on the nature of the causality between the financial accelerator and the crowding out effect.

Hence, as long as the financial accelerator signals a financial distorsion as it affects relative prices, the introduction of an additional distorsion in purview which is the crowding out effect can exert a partially offsetting effect of the negative impact of the financial acceleration on financial stability thereby leading government budget deficits financing through the credit market to exhibit beneficial effects on financial stability overall.

When crowding out effect increases credit market sector stability is deteriorated and stock market sector stability is improved through the interaction with the financial accelerator financial frictions induced which are information asymmetries. An unstable equilibrium is reached when financial stability deterioration in the banking sector is exactly offset by financial stability improvement in the stock market sector.

At this saddle path equilibrium increases in crowding out effect result in a decrease in financial accelerator that drives down the crowding out effect until the reversal effect occurs as long as decreases in crowding out effect will increase financial accelerator and drive up crowding out effect. This sequence of reversed causality effects justifies the instability that describes the Saddle path.

3. Results

3.1. The process of the Saddle equilibrium setting

When crowding out effect increases credit market sector stability is deteriorated and stock market sector stability is improved through the interaction with the financial accelerator financial frictions induced which are information asymmetries. An unstable equilibrium is reached when financial stability deterioration in the banking sector is exactly offset by financial stability improvement in the stock market sector.

At this saddle path equilibrium increases in crowding out effect result in a decrease in financial accelerator that drives down the crowding out effect until the reversal effect occurs as long as decreases in crowding out effect will increase financial accelerator and drive up crowding out effect. This sequence of reversed causality effects justifies the instability that describes the Saddle path. The effects of the relationship between the crowding out effect and the financial accelerator are threefold. One effect on the credit sector through eviction of loanable funds by what we call the cost push effect.

One effect on stock market sector or what we call the saddle path unstable equilibrium between crowding out effect and the financial accelerator. One cross effect of government bonds and securities between the credit market and the stock market sectors that makes improvement in stock market sector trading of government securities or attract ability to banking placement incur draining loanable funds through increase in stock market profitability and deteriorates credit market financial stability by the demand pull effect through the effect of liquidity risk on credit risk in the face of limited tolerance to overall risk incurred by banks and the search for yield motivation of banks.

3.2. The mathematical formula of the saddle path equilibrium

In mathematics, a saddle point or minimax point is a point on the surface of the graph of a function where the slopes (derivatives) in orthogonal directions are all zero (a critical point), but which is not a local extremum of the function. To solve the problem, we refer either to the Lagrangian optimization or the Gaussian pivot or the Hessian matrix optimization conditions for a two variables function setting the Gaussian pivot to zero or optimizing the Lagrange equation or defining the Hessian as indefinite.

According to Petresdorff, et al. [17], in the book "Critical points of autonomous systems", the name Saddle derives from the fact that the prototypical example in two dimensions is a surface that curves up in one direction then curves down in the opposite direction resembling riding a saddle or a mountain pass between two peaks forming a landform saddle. A saddle point is defined also as a point of inflection where the tangent does not cross any part of the saddle. It is also called a local extremum.

Nie, et al. [18] use a mathematical proof for saddle point problems of polynomials using Gaussian pivot and the lagragian multiplier for bivariate polynomial functions and show there exist Saddle points for bivariate polynomials. Therefore, Financial stability as has been expressed in the previous section exhibits a saddle point.

3.3. Rationale for adopting saddle path instability type equilibriums

As stated by Stemp (2006), Economic models derived from optimizing behavior are often characterized by non- linearity and saddle path instability. It is consented that economic models involving production, financial markets, information asymmetries and monopolistic competition are nonlinear in their structure. The economic modeling of the relationship between the crowding out effect and the financial accelerator involve all of these issues. It is therefore nonlinear and presumably involves saddle path instability hypothesized by the nonlinear relationship that is seemingly obvious from economic analysis as their causal relationship is inversed in sign one way the opposite of the other.

The existence of a saddle path is thus verified according to the previous expression of FA by COE which shows non-linearity. Indeed, disrespectfully of the mathematical proof, the fact that increases in the crowding out effect affect positively the financial accelerator whereas increases in the financial accelerator affect negatively the crowding out effect corroborate the hypothesis of non-linearity and provides a clear proof of the existence of jump points in equilibriums and saddle instability.

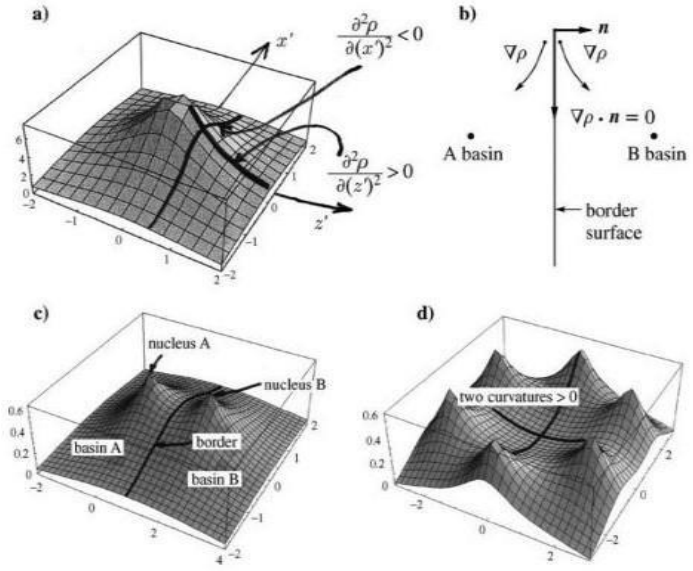

Figure 1: *Mountain pass of a Saddle point.*

Figure 1 shows a special overview of the mountain pass for the extremum of the function subjected to the saddle path and the point of inflexion of the saddle point. As will be shown later in the case study financial stability exhibits nearly the same pattern of evolution for the adopted case with multiple equilibrium points of inflexion and mountain passes.

3.4. Mountain pass theory of a sufficient condition for the existence of a saddle path

There is a certain point of inflexion in the relationship between those two figures interlinked in inverse sign of causality due to the fact that increases in crowding out effect imply increases in the financial accelerator but increases in the financial accelerator imply decreases in the crowding out effect.

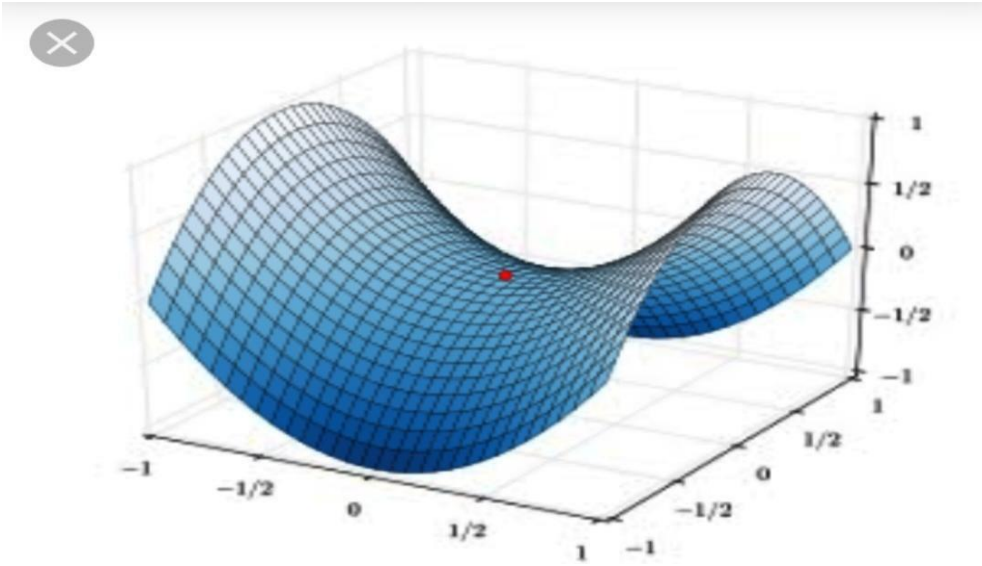

Figure 2: *A graphic representation of a Saddle point.*

3.5. Financial stability assessment through financial system risk dispersion analysis

There is obviously non-linearity between crowding out effect and financial accelerator that strengthens the hypothesis of the existence of a saddle path where the crowding out effect acts as a braking effect for increases in financial accelerator detrimental effects on financial stability.

The issue of non-linearity among constituents of financial stability assessment components has raised a debate that suspected the goodness of fit of macro-financial modeling while paving the way for the reference to general equilibrium modeling like stated in Bollen, et al. [1].

4. Discussions

4.1. Virtual economic of nonlinear relationship between crowding out effect and financial accelerator

4.1.1. The Model of virtual economy

We assume a virtual economy comprising a corporation producing goods and financing itself through borrowing from the credit sector and from equity. A credit sector with a principal activity of credit production facing the problem of maximizing its profit and imposing a borrowing rate with agency costs subjected to variable stance of information asymmetries that aggravate the financial accelerator.

Third, there is a government spending with resources coming from collecting taxes and borrowing from the credit sector to finance its expenditure. The borrowing rate is assumed to be the same disrespectfully of the risk entailed by the credit. In other words, the state borrows at the same rate as the corporation, the risk premiums are the same and only the agency costs vary.

And fourth, there is a household contributing with labor to the production of goods of the corporation and maximizing its welfare depending on income, financial stability and leisure. The household exerts an investment activity which is the placement in the corporation. The frictions coming from the government and the corporation introduce constraints on welfare maximization so that financial stability is depending on the frictions introduced by the government on investment which is the crowding out effect and the transmission of shocks from the real sector to the financial sector on financial stability with whom is concerned the household through the financial accelerator. The economy is assumed to be in autarky. Hence there are no external markets nor trading.

1. The corporation equilibrium:

The single corporation in the economic setting represents a monopoly in compliance with New Keynesian setting of imperfect competition. Its problem is to maximize profit. But we are not concerned with prices of products. Rather we are concerned with internal external financing wages and marginal products of capital and labor.

MPk=Marginal product of capital= $df(k, l)/\ dk$
MPl= Marginal product of labor= df(k,l)/dl
D= Dividends
Br=Borrowing rate
r= real interest rate
ς =depreciation rate or cost of production
Profit= MPk-(D/Br) (r-ς)+ Markup
Tobins Q theory stipulates that Q= Market value of utilized capital/Replacement cost of installed capital.
At equilibrium Q=1
Market value of utilized capital= Principal component of Stock index
Replacement value of installed capital= GDP growth

2. The government budget equilibrium assuming there are no changes in Money supply
GS= g= T+Credit public sector where t is the tax rate and C is consumption of households and g stqnds for government spending.
Y= C(y-t(y)) +i(r)+g
dy= C′(dy-t′dy)+I′dr+dg
=C′(1-t′)dy+I′dr+dg
We assume there is no monetary policy as it is beyond the scope of this research.
M/P=l(r)+k(y)
d(M/P)=l′dr+k′dy
Totally differentiating yields:
0=l′dr+k′dy as there are no changes in Money Supply
dr= -k′/l′ dy
dy= C′(1-t′)dy-i′/k′ dy +dg
dg= (1-C′(1-t′))+i′k′/l′)dy (0)

3. The credit sector equilibrium:
The credit intermediary maximizes profit:
Profit= CP*Br+Div-RR-Operating costs-Agency costs-NPLs-CP*MMR. CP=Cpub+Ccorp
RR stands for reserve requirements
NPL are non-performing loans and MMR the money market rate.
At the welfare optimization, the consumption is maximized under the constraint that financial stability is maximized. Financial stability is maximized for FA=1 and COE superior to zero.
Setting FA=1 yields Div= Br
Profit of the credit sector=CP(2Br-MMR)+-RR-(Agency costs+Operating costs+NPLs)

4. The household welfare optimization and constraints:
The household supplies labor for the corporation, consumes goods produced by the corporation and invests as a stakeholder in the corporation a proportion of his income. Its objective is to maximize consumption, income, financial stability and leisure time which we will neglect for analytical purposes and especially for the fact that as long as the horizon is infinite it is worthless to make leisure variable as there is no arbitrage between foregoing leisure to benefit from income for the coming generations.

The household welfare is a function of consumption, leisure, financial stability and income
W= f(C, L, FS, I) where welfare improves for increases in consumption, financial stability, income and leisure time. Welfare maximization assumes maximizing consumption under the constraint that financial stability and income is maximized taking Leisure as a scalar.

Max W= Max f(C, L, FS, I)= Max f*(C,FS,I)
FS is a function of crowding out and financial accelerator.

Financial stability is maximized if FA is zero as usually FA exerts a negative effect on financial stability and COE is positive in other words there is crowding in.

For household welfare maximization, we maximize consumption under the constraint that FA is zero and COE is superior to zero for FS maximization and income is maximized. Having taken leisure for scalar maximizing income is about maximizing revenue from investment. We assume the model is an infinite horizon model with a dividend distributed from the corporation for the initial investment as a placement and which is reinvested with a proportion of salaries.

So that
C+i= D+w= D+ df(k,l)/dl
Profit= MPk-(D/Br) (r-ς)+Markup
Profit is maximized for MPk equals MPl

i=in(MPk-(D/Br) (r-ς))+μK
C+ in(MPk-(D/Br) (r-ς))+μK=D+MPk
C= D+MPk- in(MPk-(D/Br) (r-ς))-μK.
The expressions of COE and FA as sensitivities give:
COE= (di/dg)*(i/g)
FA=(dPCSI/dGDPgrowth)*(GDPgrowth/PCSI) (1)
FS= Var (COE+FA)= Var(COE) +Var(FA) +2Cov(COE,FA)
Tobin's Q theory gives PCSI/GDP= Q
At equilibrium with Q=1:
PCSI=GDP=y=C+i+g (2)
GDPgrowth= β*i (3)
(1), (2) and (3) yield:
FA= (dy/dα*i)(α*i/y)
From the fiscal multiplier (0):
dg= (1-C′(1-t′))+i′k′/l′)dy
dg/dy=1-C′(1-t′))+i′k′/l′

4.1.2. A mathematical analysis of the virtual economy assertions

Through chain rule theorem:
dg/dy=(dg/di)*(di/dy).
dg/di=dy/di*(1-C′(1-t′))+i′k′/l′)=(dc/di+dg/di+1)(1-C′(1-t′))+i′k′/l′)
dy/di= d(c+i+g)/di= dc/di +dg/di +1
dg/di=(dc/di+dg/di+1)(1-C′(1-t′))+i′k′/l′) (4)
FA=(dy/dα*i)(α*i/y)
=(d(c+i+g)/dα*i)(α*i/c+i+g)
FA=1/α*(dc/di+1+dg/di) *α*i/(c+i+g)
(c+i+g)/i=FA/(dc/di+1+dg/di) (5)
(4) and (5) give dg/di+dc/di +1= FA*i/(c+i+g)
dg/di=FA*i/(c+i+g)-1-dc/di or else dc/di= FA*i/(c+i+g)-1- dg/di
Substituting into (4) yields dg/di= (FA*i/(c+i+g))*(1-C′(1-t′))+i′k′/l′)

For consumption maximization under the context of welfare optimization we maximize consumption under the constraint that FS is maximized

Financial stability is maximized when FA =1 and COE is superior to 0
Setting FA=1 yields c+i+g/i=dc/di+dg/di+1
Thus : dg/di= FA/(dc/di+dg/di+1)*(1- C′(1-t′))+i′k′/l′)
Thus dc/di*dg/di+(dg/di)2+dg/di= FA (1-C′(1-t′))+i′k′/l′)

(dg/di)2 + (FA*i/(c+i+g)-1- dg/di)dg/di+dg/di= FA (1-C′(1-t′))+i′k′/l′)
(FA*i/(c+i+g))*(dg/di)= FA (1-C′(1-t′))+i′k′/l′)
COE=di/dg*g/i
dg/di*i=g/COE
FA*g/(COE(c+i+g))=FA(1-C′(1-t′))+i′k′/l′)
COE=g/(c+i+g)(1- C′(1-t′))+i′k′/l′)
(1- C′(1-t′))+i′k′/l′)=g/((c+i+g) COE) Thus c+i+g=g/(COE*(1- C′(1-t′))+i′k′/l′)′
FA= (dy/dα*i)(α*i/y)
dy/di=dg/di+dc/di+1
dg/di=FA*i/(c+i+g)-1-dc/di (6)
dy/di=FA*i/(c+i+g)
FA= FA*i2/(c+i+g)*g*(COE*(1- C′(1-t′))+i′k′/l′))
C+i+g=i2*g*COE*(1- C′(1-t′))+i′k′/l′)

Implicit differentiation with respect to i and assuming COE is not differentiable with respect to i meaning that crowding out does not depend on the level of investment but on the level of government spending and structural effects pertaining to infrastructure and government borrowing from the credit sector.

But as investment increases the crowding out effect which is the transformation rate between the increases in government spending and the decreases in private investment does not vary. The implicit differentiation yields

dc/di +1+dg/di =2i g*COE*(1- C′(1-t′))+i′k′/l′)+i2dg/di *COE*(1- C′(1-t′))+i′k′/l′)
FA*i/(c+i+g) =2i g*COE*(1- C′(1-t′))+i′k′/l′)+i2dg/di *COE*(1- C′(1-t′))+i′k′/l′)
dg/di= (FA*i/(c+i+g))*(1-C′(1-t′))+i′k′/l′)
FA*i/(c+i+g)=2ig*COE*(1-C′(1-t′))+i′k′/l′)+i2(FA*i/(c+i+g))*(1-C′(1-t′))+i′k′/l′)2*COE
FA((i- 2i4)/(c+i+g))*(1-C′(1-t′))+i′k′/l′)2 *COE)=2ig*COE*(1-C′(1-t′))+i′k′/l′)
Hence, we get the expression FA(a+bCOE)=cCOE hence FA=cCOE/(a+bCOE)
Hence the relationship between FA and COE is nonlinear.
Therefore Cov (COE, FA) =0.

4.2. Financial stability formulation, the adopted equation

As documented in the literature for instance from the Bank of International Settlement as stated in Gadanecz, et al. [19], Financial stability is measured as stress computed as a deviation from the historical trend.

Kaminsky, et al. [20] Demirguc-Kunt, et al. [21] used a PROBIT model to estimate an early warning system of banking instability which is a qualitative response model with two outcomes unlikely to accurately assess neither financial stability nor report a certain pattern of unstable equilibrium like the Saddle path that helps fathom the equilibrium between variables non linearly correlated. Some references compute a banking stability index using

weighted average of sub-indicators of banking sector soundness including capital adequacy, profitability, balance sheet, liquidity, asset quality, credit and currency risk. The constructed index of financial ratios is expressed in terms of the standard deviation from the historical trend.

In the following, we propose a novel method to compute it as being a dispersion of the sources of risk or the risk variance entailed by the depicted essential sources of risk to financial stability in the autarky context of the virtual economy modeling the crowding out effect and the financial accelerator. The adopted formulation takes ceteris paribus other features of the financial sector affecting financial stability and is intended to fathom the specificities of the Saddle path equilibrium that highlights the commingled effects of the crowding out effect and the financial accelerator on financial stability.

Henceforth, other features affecting financial stability can be taken for a constant affecting financial stability exogenously to the frictions being analyzed under the purview of this research. By a way of consequence, they do not affect the adopted method of variance as long as Var(C+Random variable)= Var(Random variable)

The rationale lying behind this choice is that crowding out and the financial accelerator which are main items of financial stability setting are originating from risk entailed by eviction and the transmission of real economic shocks to financial shocks which might vary in intensity according to the situation can be contained partially and have a certain dispersion in possibilities in a point in time fashion not necessarily through time.
Even in a point in time there might be several possibilities to take account of in the dispersion figure or the variance.

COE and FA are taken for risks which are the risk of eviction of private investment that affect the pricing of assets traded through investment in the stock market and by a way of consequence financial stability and the risk of propagation of real economic shocks to the financial sector thereby affecting financial stability. The rationale is that they are undesirable events with dispersed distribution and could be either excessive or tolerable depending on their intensity. Financial stability is not necessarily taken for autocorrelated as basically financial crises erupt abruptly and early warning systems although broadly adopted have not shown accuracy for crises forecastle as they usually hit unexpectedly. It is very dissented and least likely that financial stability is an autoregressive process in time series. Serial correlation assumes predictability which is not likely for financial crises eruption.

We assume that it is dispersed in crowding out effect and financial accelerator items so that it is computed as a punctual dispersion of the distribution of risk entailed by COE and FA.
FS= Var(COE+FA)= Var(COE) + Var(FA) + 2Cov(COE,FA)
We have to compute Cov(COE, FA) in order to formulate FS.

As shown in the mathematical proof of the model, we find a non-linear relationship between crowding out effect and the financial accelerator.
Therefore, Cov(COE,FA) does not exist and is taken for 0.
Which means that FS= Var(COE) +Var(FA).
The variance of these risks is as will be shown expressible as a polynomial function of their expectation which is equal to their expression.
As they are not random their expressions exactly equal to their value so that
FS= Var(COE+FA) = Var(COE) +Var(FA) is a polynomial and hence likely to verify saddle path hypotheses.

4.3. Variance of risk and the adopted formula for financial stability

We begin with the basic definition of variance of risk assuming risk is a random variable. Var (r) = Exp ((r)2)- (Exp (r))^2
And an extrapolation from the definition of Profitability Gap
Profitability Gap = Potential profitability – Actual profitability
= (Effectively likely risk)* (Measure of risk)

Assuming potential profitability is the remuneration of overall risk effectively likely risk is equal to (Risk –uncertainty).
As long as:
Risk = uncertainty + effectively likely risk
Now the measure of risk is:
The cumulation of risk pertaining to credit activity, risk pertaining to excess volatility of stock markets and projects risk which is profitability gap.

Assuming μ is the risk return tradeoff coefficient which is not necessarily scalar

This leads to Profitability gap= (Risk- Uncertainty) * ((Agency costs-Risk of NPLs) +(Stock market index-principal component of SMI)+ μ Profitability gap)

Risk = Porf gap *(1/((Agency costs-Risk of NPls)+ (Stock market index-principal component of SMI) +μ Prof gap)) + Risk Uncertainty

The expectation of profitability gap is a certain scalar α multiplied by the expectation of risk
The expectation of the multiplier for prof gap is 1/ (1+α*μ* Exp (Risk))
Exp (R)= Var (R) + α*Exp (R)/(1+α*μ*Exp(R))
Var (R) = E (R) - α*E (R)/(1+α*μ*E(R)) (0)
Var (R) = E(R)2 – (E(R))^2 (1)
E (R)^2 = Var (R) + (E(R))^2
= Var (R) + (Var (R) + αE (R)/(1+αμE(R)))^2
= Var (R) (1+ Var (R) +2 αE (R)/(1+αμE(R)) + α^{2}E^2 (R)/(1+αμE(R))^2

Substituting into (1) gives
Var (R) = Var (R) (1+ Var (R) +2 αE (R)/(1+αμE(R)) + α^{2}E^2 (R)/(1+αμE(R))^2 - E^2(R)
= Var (R) (1+ Var (R) +2 αE (R)/(1+αμE(R)) + α^2-1- α^{2}μ^{2}E^2(R)- 2 αμE(R)/(1+ α^{2}μ^{2}E^2(R)+ αμE(R)))

Setting expressions of Var R on the left-hand side yields
Var(R)* (1-1-Var (R)-2αE(R)/(1+αμE(R))=E^2(R)* (α^2-1- α^{2}μ^{2}E^2(R)- 2αμE(R)/(1+ α^{2}μ^{2}E^2(R)+ αμE(R))
-Var^2(R)= 2Var (R) αE(R)/ (1+αμE(R)) + E^2(R)* (α^2-1- α^{2}μ^{2}E^2(R)- 2αμE(R)/(1+ α^{2}μ^{2}E^2(R)+ αμE(R))
Therefore
Var(R)= -2 αE(R)/ (1+αμE(R)) - E^2(R)/Var(R)* (α2-1- α^{2}μ^{2}E^2(R)- 2αμE(R)/(1+ α^{2}μ^{2}E^2(R)+ αμE(R))
Var (R) = E (R) - αE (R)/(1+αμE(R))= E(R)(1- α/(1+αμE(R)))

The second expression leads to
E(R)(1- α/(1+αμE(R))) +2 αE(R)/ (1+αμE(R)) =- E^2(R)/Var(R)* (α^2-1- α^{2}μ^{2}E^2(R)- 2αμE(R)/(1+ α^{2}μ^{2}E^2(R)+ αμE(R))

Therefore
$E(R)\ (1+ \alpha/\ (1+\alpha\mu E(R)) = -\ (E^2(R)/\ (E(R)(1-\ \alpha/\ (1+\alpha\mu E(R))) * (\alpha^2\text{-}1-\ \alpha^2\mu^2E^2(R)-\ 2\alpha\mu E(R))/(1+\ \alpha^2\mu^2E^2(R)+\ \alpha\mu E(R))$
Simplifying both sides by E(R) leads to
$1+\ \alpha/\ (1+\alpha\mu E(R)) = (1/(1-\ \alpha/\ (1+\alpha\mu E(R)))^* (\alpha 2\text{-}1-\ \alpha 2\mu 2E2(R)-\ 2\alpha\mu E(R))/(1+\ \alpha 2\mu 2E2(R)+\ \alpha\mu E(R))$

Now we make use of the notion of implicit differentiation with respect to E(R) both sides The first term in the right-hand side is denoted by f and the second term by g
The development of the implicit differentiation leads to
$-\ \alpha 2\mu/(1+\alpha\mu E(R))^2= (f'g-fg')$ with respect to E(R)
$-\ \alpha 2\mu/(1+\alpha\mu E(R))^2 = (1/Var2(R))^* (2\ \alpha^2\mu^2E(R) +2\alpha\mu)(1+\ \alpha\mu E(R)-\ \alpha)\ (1+\ \alpha^2\mu^2E^2(R)+2\ \alpha\mu E(R))-(1-\ \alpha+\ \alpha^2\mu^2E^2(R)+2$
$\alpha\mu E(R))(\ (\alpha^2\mu+\ \alpha\mu E(R)+(1+\ \alpha\mu E(R)-\ \alpha)(2\ \alpha^2\mu^2E^2(R)+\ \alpha\mu))/(1+\ \alpha\mu E(R))$

with introducing equation (0), Var(R) = E(R) – E(R)/(1+αμE(R)) to the right-hand side and substituting by Var(R) We take $Var^2(R)$ to the numerator of the left-hand side and we develop the right-hand side for E(R) then we replace Var(R) by y and E(R) by x

We find
$Y^2 = 1/\ (\alpha^2\mu\ x)^* (x^4(2\ \alpha^5\mu^5 + 2\ \alpha^4\mu^4\text{-}2\ \alpha^4\mu^4(2-\ \alpha)) +x^3(2\ \alpha^4\mu^4(1-\ \alpha\mu)\text{-}2\ \alpha^4\mu^4 +2\ \alpha^3\mu^3) +x^2(2\ \alpha^3\mu^2(1-\ \alpha\mu)\text{-}2\ \alpha^3\mu^3\text{-}2\ \alpha^3\mu^3(2-\ \alpha)\text{-}2\ \alpha^2\mu^3(\alpha^2\text{-}\alpha) + \alpha^2\mu^2(1+2\alpha\mu)) +x(-4\ \alpha\mu^2(\alpha^2\text{-}\alpha)+\ \alpha\mu(1+2\ \alpha\mu))+2\ \alpha\mu(1-\ \alpha)\text{-}2\mu(\alpha^2\text{-}\alpha).$

By using implicit differentiation once again with respect to x we find
d(y2/dx)= 2 d(y/x) y
d(y/x) =-2x
because Var(R) = E(R2)-E2(R) hence dVar(R)/dE(R)= -2E(R) as dE2(R)/d(E(R)=0 assuming $E(R^2)$ is independent of E(R)
hence $-4xy =1/(8\alpha^2\mu x^2)^*(x^4\ A+x^3\ B +x^2C +xD +E)\text{-}1/(4\ \alpha^2\mu x^2)^*(x^3\ 4A+x^2\ 3B+x\ 3C+D)$

Therefore, simplifying the above equation gives a polynomial function of fixed coefficients with respect to E(R) and various powers.

Hence, we have shown by a mathematical proof that the variance of risk is a polynomial function of the expectation of risk

Yu, et al. [22] stated that: “Polynomial distributions provide a superior flexibility over other canonical distributions, albeit at a cost of large number of parameters and the support interval is constrained to a finite range of values. The main advantage of polynomial distributions is that they can yield parameterized closed form mathematical expressions as well as offer a much greater flexibility in modeling time evolution of probability distribution for example when describing causal interactions in complex systems and modeling state transitions in dynamic systems”

The concise fathoming of risk distribution helps proceed to comprehensive early warning like in the literature about financial stability computation for the sake of making risk management serve for financial instability mitigation and ensure better resilience from the onslaught of financial meltdowns under the scope of better shaped prudential surveillance schemes. Hence FS is a bivariate function FS=f(P(x),P(y)) which is additional and yields a bivariate function having a saddle path.

One major merit of the adopted methodology is to gather both systemic and systematic threats from a macro financial perspective and to check crook effects within the assessment process. The bank specific factors have been neglected because not only prudential surveillance is assumed to smooth them out across time but also because the key features is concerning investment that draws a wedge across financial sectors and the macroeconomic situation passing through the agency costs and the financial accelerator. Those features are implicitly invocated within the framework of the assessment of the crowding out effect and the financial accelerator that measured the impact effect of the macroeconomic overall situation on the financial system. The novelties of this assessment technique besides the fact that it is sensitivity based is that it provides clear insights about the cross effects of the constituent items taken into account in financial stability assessment and that risk taken into consideration is measured based on a comprehensive method that takes into account of the risk moments and comes up with a pattern for risk behavior that feeds the key methodological approach upon which the methodology of financial stability measurement is comprehended.

Hence polynomial distribution entailed by the bank specific risk assessment leads to a saddle path type of cross linkage among the eviction and the financial accelerator. Then the variance of the joint assessment is assumed to provide an assessment of financial stability where the use of variance clearly indicates a paramount role played by abrupt events grasped by volatility.

5. Verification of the theoretical assertions through an empirical case study on Tunisia

The analysis spans from 1983 to 2017 and tries to plot the sensitivities that have been invocated in the theoretical discussion using the elasticity formula applied to aggregates of the Tunisian economy during the same period.

$$e(t)(PI/GS)= (\Delta PI(t)/\Delta GS(t))*(GS(t)/PI(t))$$

The crowding out effect of Tunisia distinguished by frequent episodes of crowding in, meaning that the eviction has the reverse effect of stimulating private investment rather than slowing it down, it will be shown that even in this case the principal still holds and the modeling of financial stability still exhibits mountain passes and points of inflexion like predicted by the theoretical discussions displayed in the analysis.

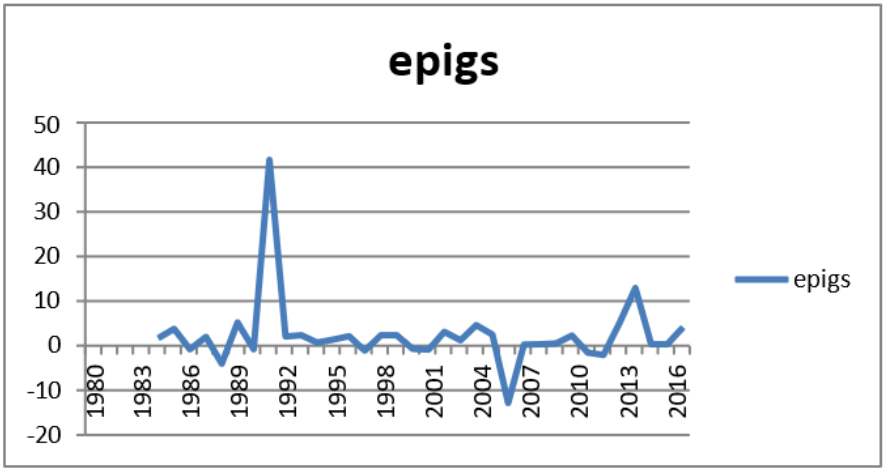

Figure 3: *Descriptive statistics of crowding out effect.*

A piking financial accelerator as shown in the figure between 1995 and 2000 is a bad signal of excessive transmission of real economic shocks to financial instability and reveals alarming as improvements in economic performance result in financial instability.

If X is crowding out e(PI/ GS)

and Y is the financial accelerator e(Principal component of Stock market Index/GDP growth) Here the principal component of the stock market index is the inverse of the investor risk appetite.

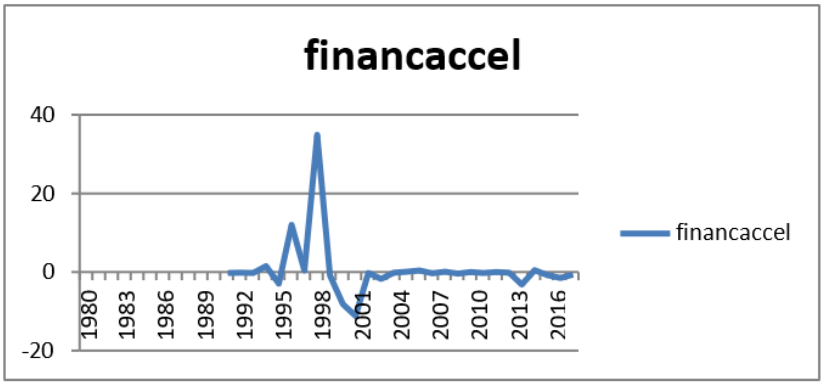

Figure 4: *Descriptive statistics of the financial accelerator.*

we have collected data from 1980 to 2019 on investor risk appetite we revert the investor risk appetite and take the sensitivity to GDP growth to get the financial accelerator for the given period.

Then Z as defined earlier is a saddle path surface which is indicated in blue in the chart. We have the Y, X as being the financial accelerator and the crowding out effect.

FS= Var(COE)+Var(FA) FS=f(P(x),P(y))
Such that FS=P(x)+P(y)=0.0002 x3-32x2+48x+15 + 0.7y2+10y+28.
It is verified in the following charts that there exists at least one saddle point which corroborates the mathematical proof and the economic analysis that have been conducted all along this article.

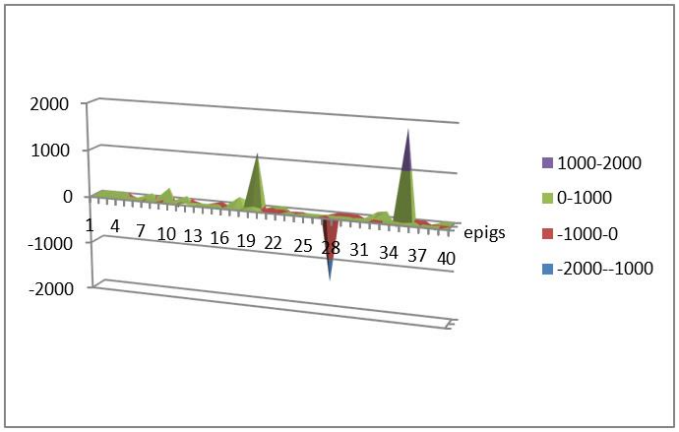

Figure 5: *Saddle path plotting in three-dimension graph and the prevalence of a mountain pass.*

The above figure 5 shows two to three saddle paths easily depictable from sight and are even better depictable from the radar disposition that shows clearly the points of inflexion.

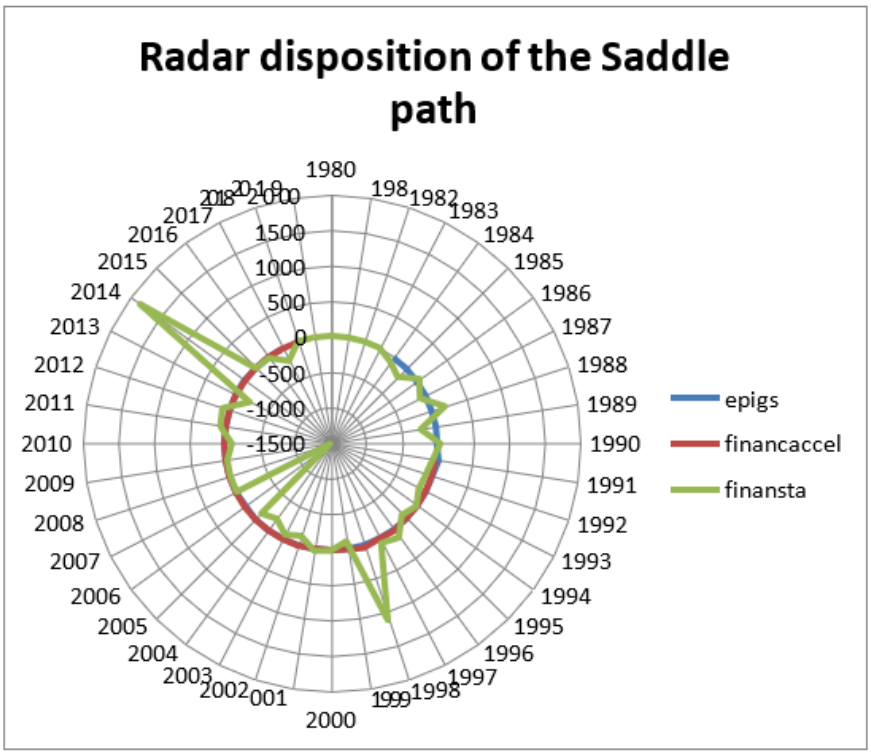

Figure 6: *The radar disposition of the Saddle path and the reiterating pikes and troughs and Saddle points for financial stability.*

The theoretical assertions concerning the mountain pass of the saddle path, the polynomial distribution of risk and the validity of the sensitivity analysis have been verified. This corroborates the theoretical analysis and materializes a case study of a small open economy although the theoretical predilections are assumed to hold even for developed countries because the analysis has been conducted disrespectfully of the level of development of the country and focusing on key interactions among fundamental features of financial stability and systemic risk features.

6. Conclusion

The main economic implication of the existence of a saddle path between crowding out and the financial accelerator is that there is no self-sustaining way to improve financial stability although in critical zones it is assumed to reestablish a certain pattern. When credit market financial stability is improved stock market sector financial stability is deteriorated. Besides, the crowding out effect smoothest out the negative effects of financial frictions. Financial frictions mean financial constraints that prevent corporations from finding all desirable investments from external sources.

These include information asymmetries.

In definitive what mitigates financial instability results in slowdown of economic performance provided there is no crowding in like in the adopted case study.

In imperfect markets when the economic system of the market economy is subject to distorted phenomena and markets financial instability should be mitigated by cautious additions of further distortions in order to benefit from the externalities of the simultaneous prevalence of several distortions like for new Keynesian economic growth prospects or the double dividend theory where the first distortion impedes efficiency. But additional distortions improve efficiency through partial offsetting of the detrimental effect of the initial distortion. This is the way crowding out effect affects positively financial stability even though it is claimed to deteriorate. Basically, once established that the equilibrium between the financial accelerator and the eviction effect is unstable and generated a mountain pass in what we call a saddle surface.

We should act on financial stability in the phase were crowding out effect plays like a brake for the financial accelerator deterioration of financial stability. If the financial accelerator is of commensurate importance in the face of prevalence of important information asymmetries accentuating the agency costs in the banking sector there should be more room left for budget deficit financing through banking credit in order to benefit from the double dividend effect. In fact, for the case study at hand in Tunisia there are several information asymmetries like free riding and adverse selection that increase the agency costs and the average borrowing rate tremendously and budget deficits are financed through credit borrowing excessively in the face of shortage of other funding sources of budget deficit financing as the central banks abides money printing.

This is assumed to be detrimental for the financial system. But it does attenuate financial instability considerably. It has been shown from the case study of Tunisia crowding out effect and financial accelerator that the mountain pass of the saddle path does effectively exist. This theory advocates the merits of government frictions in new Keynesian economic arena in comparison to a neoclassical one and shows that in a situation of imperfect competition and sticky prices efficiency can improve through addition of frictions and distortions. When we express financial stability in terms of the dispersion of risk of crowding out effect and financial acceleration we find theoretical evidence of a Saddle path for behavior of financial stability with local minimaxes in compliance with theory on the Saddle path of unstable equilibriums. The new Keynesian framework is a good fit of an economy describing what occurs across the real economy and the financial sector with realistic paradigmatic assumptions about imperfect competition and imperfect information disrespectfully of staggered prices and habits formation for consumption which although obviously affecting the setting of the saddle path is of secondary importance compared with private investment government spending and the financial accelerator.

In order to be cautious about financial stability the economy has to be located underneath the saddle point for financial stability to be at its maximum in the mountain pike. For this sake caution has to be taken about the crowding out effect when the policy maker decides to increase government spending to conduct a fiscal stimulation. The double dividend theory is verified under this context and corroborates the idea that adding frictions when they are already prevailing can improve the situation through attenuation of the negative effect of the first frictions especially in the New Keynesian framework based on imperfect competition and credit markets and staggered prices. Henceforth, this analysis adds up to the available literature on financial stability assessments methods while attempting not to wave merits of available methods it a try to complement them by using state of the art mathematical tools that help circumvent the hindrances of past attempts to provide an assessment of financial stability even though cross interactions among features composing financial stability expressions were graspable with difficulty.

The likely limitations of this research are about the role played by the issue of monetary policy although that is closely related to the objective of financial stability which recently is dedicating a vast amount of research about prudential surveillance having for objective of containing credit risk and liquidity risk for the purpose of dampening financial instability through macro-prudential and micro-prudential restrictions as thoroughly discussed within the scope of the Basel frameworks that add up impetus to the value at risk analysis of financial stability.

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